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through the sieve of unconsciousness. But there is always a succession of voluntary efforts which has put the whole machine into motion and offers favorable combinations.

The eminent geometrician is entirely right, and I can not protest too strongly against the tendency of certain psychologists to exaggerate the rôle of the unconscious after it had been too much neglected. Distrust of our reason must not lead us to yield everything to instinct; nor the scorn of clear thought, to imagine any sort of obscure intelligence.

It is now many years since I myself pointed out "the positive orientation which serves as a good preparation for our system of images,"[5] and noted that this orientation presupposed a "choice."[6] I was speaking of the work of the artist and poet, but are there not the closest relations between our most widely diversified modes of creation?

Have we not all observed also what happens when we have left a task in what I shall call a state of confusion (*embrouillement*)? When we resume it we no doubt gain by the comparative rest accorded our nervous cells, but only the salient ideas, the principal points of view have survived and have taken their rank in the confused mass of our thoughts. The task already begun has been continued in our mind; a sort of purification has taken place and one last effort is enough to bring it to the point. So, borrowing M. Poincaré's comparison, the gaseous atoms put in motion by preliminary effort continue their dance after our will no longer has control, but in the direction in which it has impelled them.

If we mean that work is accomplished without us, at least it has always been prepared by us. There is a clear state of consciousness in the finishing as well as at the start.

LUCIEN ARRÉAT.

PARIS, FRANCE.

FOUR-FOLD MAGICS.

Having read Professor Kingery's interesting article in the April *Monist*, entitled "Magic in the Fourth Dimension," I wish

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but in a case of this kind visual attractiveness is affected, if I may say so, by intellectual attractiveness.

[5] *Memoire et imagination*, p. 134.

[6] *Art et psychol. indiv.*, p. 122.

to make some comments which may be of general interest.[1] In my opinion his treatment does not clearly argue out the analogy, with the result that none of his productions can be considered as a fourfold magic.

A magic square has two magic directions parallel to its sides through any cell—a row and a column; a magic cube has three magic directions parallel to its edges, a row, a column and a "line," the latter being measured at right angles to the paper-plane. By analogy, if for no other reason, a magic 4-fold should have four magic directions parallel to its linear edges, a row, a column, a line, and an "*i*." [The *i* is a convenient abbreviation for the imaginary direction, after the symbol $i=\sqrt{-1}$.] It is quite easy to determine by analogy how the imaginary direction is to be taken. If we look at a cube, set out as so many square sections on a plane, we see that the directions we have chosen to call rows and columns are shown in the square sections, and the third direction along a line is found by taking any cell in the first square plate, the similarly situated cell in the second plate, then that in the third and so on. In an octrahedroid the rows, columns and lines are given by the several cubical sections, viewed as solids, while the fourth or imaginary direction is found by starting at any cell in the first cube, passing to the corresponding cell of the second cube, then to that of the third, and so on. If we examine Professor Kingery's examples, we see that only his octahedroids of orders $4n$, i. e., 4^4, 8^4, are magic in the imaginary direction.

If we denote each of the nine subsidiaries of order 3 in Fig. 1 by the number in its central cell, and take the three squares 45, 1, 77, in that order, to form the plates of a first cube; 73, 41, 9 to form a second cube, and 5, 81, 37 for a third cube, we get an associated octahedroid, which is magic along the four directions parallel to its edges and on its 8 central hyperdiagonals. Comparing the above figure with Professor Kingery's Fig. 1, which is only "semi-nasik," we find the magic sum on 9 rows, 9 columns and 18 diagonals, the nine subsidiaries equally weighted and magic in rows and columns, and further the square is 9-ply, that is the nine numbers in *any* square section of order 3 give the magic sum of the great square.

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[1] The subject has been treated before in:

Frost (A. H.), "The Properties of Nasik Cubes," *Quarterly Journal of Mathematics,* London, 1878, p. 93.

"C. P." (C. Planck), "Magic Squares, Cubes, etc.," *The English Mechanic,* London, March 16, 1888.

Arnoux (Gabriel), *Arithmétique graphique,* Paris, 1894, Gauthier-Villars et Fils.

Planck (C.), *The Theory of Path Nasiks,* 1905. Printed for private circulation. There are copies at the British Museum, the Bodleian, Oxford, and the University Library, Cambridge.

It will be convenient here to turn aside and examine the evolution of the nasik idea and the general analogy between the figures of various dimensions in order that we may determine how the nasik concept ought to be expanded when we apply it in the higher dimensions. This method of treatment is suggested by Professor Kingery's remark, p. 310, "It is not easy—perhaps it is not possible—to make an absolutely perfect cube of 3." If we insist on magic central diagonals we know that, in the restricted sense, there is only one magic square of order 3, but if we reckon reflections and reversions as different there are 8. If we insist on magic central great diagonals in the cube, as by analogy we ought to do, then, in the restricted sense, there are just 4 magic cubes of order 3. But each of these can be placed on any one of six bases and then viewed from any one of four sides, and each view thus obtained can be duplicated by reflection. In the extended sense, therefore, there are 192 magic cubes of order 3. None of these, however, has the least claim to be considered "perfect." This last term has been used with several different meanings by various writers on the subject. From the present writer's point of view the nasik idea, as presently to be developed, ought to stand pre-eminent; next in importance comes the ply property, then the adornment of magic subsidiaries, with the properties of association, bent diagonals of Franklin, etc., etc., taking subordinate places.

65	6	52	29	78	16	20	42	61
36	73	14	27	37	59	72	1	50
22	44	57	67	8	48	31	80	12
69	7	47	33	79	11	24	43	56
28	77	18	19	41	63	64	5	54
26	39	58	71	3	49	35	75	13
70	2	51	34	74	15	25	38	60
32	81	10	23	45	55	68	9	46
21	40	62	66	4	53	30	76	17

Fig. 1.

The lattice idea certainly goes back to prehistoric time, and what we now call the rows and columns of a rectangular lattice first appealed to man because they disclose contiguous rectilinear series of cells, that is sets of cells, whose centers are in a straight line, and each of which has linear contact with the next. It must soon have been noticed that two other series exist in every square, which fulfil the same conditions, only now the contact is punctate instead of linear. They are what we call the central diagonals. It was not until the congruent nature of the problem was realized that it became apparent that a square lattice has as many diagonals as rows and columns together. Yet the ancient Hindus certainly recognized this congruent feature. The eccentric diagonals have been called "broken diagonals," but they are really not broken if we remember that we tacitly assume all space of the dimensions under consideration saturated with contiguous replicas of the figure before us, cells similarly situated in the several replicas being considered identical. A. H. Frost[2] nearly 50 years ago invented the term "nasik" to embrace that species of square which shows magic summations on all its contiguous rectilinear series of cells, and later extended the idea by analogy to cubes,[3] and with less success to a figure in four dimensions. If the nasik criterion be applied to 3-dimensional magics what does it require? We must have 3 magic directions through any cell parallel to the edges, (planar contact), 6 such directions in the diagonals of square sections parallel to the faces (linear contact), and 4 directions parallel to the great diagonals of the cube (point contact), a total of 13 magic directions through every cell. It has long been known that the smallest square which can be nasik is of order 4, or if the square is to be associated, (that is with every pair of complementary numbers occupying cells which are equally displaced from the center of the figure in opposite directions), then the smallest nasik order is 5. Frost stated definitely[4] that in the case of a cube the smallest nasik order is 9; Arnôt[5] was of opinion that it would be 8, though he failed to construct such a magic. It is only quite recently[6] that the present writer has shown that the smallest nasik order in k dimensions is always $2k$, (or $2k+1$ if we require association).

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[2] *Quarterly Journal of Mathematics,* London, 1865, and 1878, pp. 34 and 93.

[3] The idea of the crude magic cube is, of course, much older: Fermat gives a 4^3 in his letter to Mersenne of the 1st of April, 1640. *Œuvres de Fermat,* Vol. II, p. 191.

[4] *Quarterly Journal,* Vol. XV, p. 110.

[5] *Arithmétique graphique,* Paris, 1894, p. 140.

[6] *Theory of Path Nasiks,* 1905.

It is not difficult to perceive that if we push the nasik analogy to higher dimensions the number of magic directions through any cell of a k-fold must be $\frac{1}{2}(3^k-1)$, for we require magic directions from every cell through each cell of the surrounding little k-fold of order 3. In a 4-fold nasik, therefore, there are 40 contiguous rectilinear summations through any cell. But how are we to determine these 40 directions and what names are we to assign to the magic figures in the 4th and higher dimensions? By far the best nomenclature for the latter purpose is that invented by Stringham,[7] who called the regular m-dimensional figure, which has n $(m-1)$-dimensional boundaries, an m-fold n-hedroid. Thus the square is a 2-fold tetrahedroid (tetragon), the cube a 3-fold hexahedroid (hexahedron); then come the 4-fold octahedroid, the 5-fold decahedroid, and so on. Of course the 2-fold octahedroid is the plane octagon, the 3-fold tetrahedroid the solid tetrahedron; but since the regular figure in k dimensions which is analogous to the square and cube has always $2k$ $(k-1)$-dimensional boundaries—is in fact a k-fold $2k$-hedroid—the terms octahedroid, decahedroid, etc., as applied to magics, are without ambiguity, and may be appropriately used for magics in 4, 5, etc. dimensions, while retaining the familiar "square," "cube," for the lower dimensions.

To obtain a complete knowledge of these figures, requires a study of analytical geometry of the 4th and higher dimensions, but, by analogy, on first principles, we can obtain sufficient for our purpose. If we had only a linear one-dimensional space at command we could represent a square of order n in two ways, ("aspects"), either by laying the n rows, in order, along our linear dimension, or by dealing similarly with the n columns. In the first aspect, by rows, the cells which form any column cannot appear as contiguous, though they actually are so when we represent the figure as a square on a plane. Similarly we can represent a cube on a plane in three aspects. Suppose the paper-plane is placed vertically before us and the cube is represented by n squares on that plane (P-plane aspect). We get a second aspect by taking, in order, the first column of each square to form the first square of the new aspect, all the second columns, in order, to form the second square of the new aspect, and so on (V-plane aspect). We obtain a third aspect by dealing similarly with the rows (H-plane aspect). Here the "lines," which appear as contiguous cells in the V- or H-plane aspects do not so appear in the P-plane aspect, though they actually are contiguous when we examine the cube as a solid in three dimensions. Now consider an octahedroid represented by *n* cubes in a space of three dimensions. We get a second aspect by taking the *n* anterior, vertical square plates of each cube, in order, to form a first new cube; the *n* plates immediately behind the anterior plate in each cube to form a second new cube, and so on. Evidently we obtain a third aspect, in like manner, by slicing each cube into vertical, antero-posterior plates, and a fourth aspect by using the horizontal plates. Carrying on the same reasoning, it becomes clear that we can represent a *k*-fold of order *n,* in *k*—1 dimensions, by *n* (*k*—1)-folds, in *k* dif-

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[7] *American Journal of Mathematics,* Vol. III, 1880.

34	74	15	65	6	52	24	43	56
23	45	55	36	73	14	64	5	54
66	4	53	22	44	57	35	75	13
20	42	61	33	79	11	70	2	51
72	1	50	19	41	63	32	81	10
31	80	12	71	3	49	21	40	62
69	7	47	25	38	60	29	78	16
28	77	18	68	9	46	27	37	59
26	39	58	30	76	17	67	8	48

Fig. 2. P_1- and P_2-aspects.

69	20	34	7	42	74	47	61	15
28	72	23	77	1	45	18	50	55
26	31	66	39	80	4	58	12	53
25	33	65	38	79	6	60	11	52
68	19	36	9	41	73	46	63	14
30	71	22	76	3	44	17	49	57
29	70	24	78	2	43	16	51	56
27	32	64	37	81	5	59	10	54
67	21	35	8	40	75	48	62	13

Fig 3. V-aspect.

69	7	47	28	77	18	26	39	58
20	42	61	72	1	50	31	80	12
34	74	15	23	45	55	66	4	53
25	38	60	68	9	46	30	76	17
33	79	11	19	41	63	71	3	49
65	6	52	36	73	14	22	44	57
29	78	16	27	37	59	67	8	48
70	2	51	32	81	10	21	40	62
24	43	56	64	5	54	35	75	13

Fig. 4. H-aspect.

ferent aspects. Thus we can represent a 5-fold decahedroid of order *n*, in 4-dimensional space, by *n* 4-fold octahedroids, and this in 5 different ways or aspects.

Return now to Fig. 1 and the rule which follows it, for forming from it the magic octahedroid of order 3. If we decide to represent the three cubic sections of the octahedroid by successive columns of squares we get Fig. 2.

If we obtain a second aspect by using the square plates of the paper-plane, as explained above, we find that this is equivalent to taking the successive rows of squares from Fig. 2 to form our three cubes, instead of taking the columns of squares. Thus the presentation plane shows two different aspects of an octahedroid; this is due to the fact that the fourth dimension is the square of the second. We may call these aspects P_1- and P_2-aspects. The aspect obtained by using antero-posterior vertical planes is shown in Fig. 3, that from horizontal planes in Fig. 4. We may call these the V- and H-aspects. If we use the rows of squares in Figs. 3 or 4 we get correct representations of the octahedroid, but these are not new aspects, they are merely repetitions of P_1, for they give new views of the same three cubes as shown in P_1. In the same way, if we turned all the P-plane plates of a cube upside down we should not call that a new aspect of the cube. The aspects P_2, V, H can be obtained from P_1 by turning the octahedroid as a whole in 4-dimensional space, just as the V-plane and H-plane aspects of a cube can be obtained from the P-plane aspect by turning the cube in 3-dimensional space. Fig. 4, above, is Fig. 2 turned through a right angle about the plane of xy; we can turn about a plane in 4 dimensions just as we turn about a straight line in 3 dimensions or about a point in 2 dimensions. It will be noticed that in the four aspects each of the 4 directions parallel to an edge becomes in turn imaginary, so that it cannot be made to appear as a series of contiguous cells in 3-dimensional space; yet if we had a 4-dimensional space at command, these four directions could all be made to appear as series of contiguous cells. There is one point, however, which must not be overlooked. When we represent a cube as so many squares, the rows and columns appear as little squares having linear contact, but actually, in the cube, the cells are all cubelets having planar contact. Similarly, in an octahedroid represented as so many cubes the rows and columns appear as cubelets having planar contact, but in the octahedroid the cells are really little octahedroids having solid, 3-dimensional contact.

When we examine the above octahedroid (Figs. 2-4) in all its aspects we see that there are through every cell 4 different directions parallel to the edges, 12 directions parallel to the diagonals of the square faces, and 16 directions parallel to the great diagonals of the several cubical sections. There remain for consideration the hyperdiagonals, which bear to the octahedroid the same relation that the great diagonals bear to a cube. If we represent a cube by squares on a plane we can obtain the great diagonals by starting at any corner cell of an outside plate, then passing to the next cell of the corresponding diagonal of the succeeding plate, and so on. Similarly we obtain the hyperdiagonals of the octahedroid by starting from any corner cell of an outside cube, passing to the next cell on the corresponding great diagonal of the succeeding cube, and so on. Evidently there are 8 central hyperdiagonals, for we can start at any one of the 8 corners of one outside cube and end at the opposite corner of the other outside cube. There are therefore, through any cell, 8 different directions parallel to the central hyperdiagonals. With the directions already enumerated this makes a total of 40 directions through each cell and agrees with the result already stated.[8] Evidently the number of k-dimensional diagonals of a k-fold is 2^{k-1}, and if the analogy with the magic square is to be carried through then all the central k-dimensional diagonals of a k-fold ought always to be magic. It is here that Professor Kingery's 4^4 and 8^4 fail; they are not magic on their central hyperdiagonals.

The smallest octahedroid which can have all these 40 directions magic is 16^4, and the writer has given one of the 256 square plates of this magic and a general formula by which the number occupying any specified cell can be determined. But it will be interesting to determine how nearly we can approach this ideal in the lower orders. The octahedroid of order 3 can be but crude, and practically Fig. 2 cannot be improved upon. All rows, columns, lines, and "*i*"s are magic, and likewise the 8 central hyper-diagonals. Of course, since the figure is associated, all central rectilinear paths are magic, but this is of little account and other asymmetrical magic diagonal summations are purely accidental and therefore negligible.

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[8] If we call the diagonals in square sections parallel to faces 2-dimensional, those parallel to the great diagonals of cubical sections 3-dimensional, etc., etc., then the number of m-dimensional diagonals of a k-fold is $2^{m-1}k!/m!(k-m)!$ In fact the number required is the $(m+1)$th term of the expansion of $\frac{1}{2}(1+2)^k$. It will be noticed that this reckons rows, columns etc. as "diagonals of one dimension."

Turning to the next odd order, 5: Professor Kingery's Fig. 2 is not a magic octahedroid as it stands, but a magic can be obtained from it by taking the diagonals of subsidiary squares to form the 5 cubes. Denoting each subsidiary by the number in its central cell, we may use 602, 41, 210, etc. for the first cube; 291, 460 etc. for the second cube; 85, 149, etc. for the middle cube, etc., etc. But few of the plane diagonals through any cell of this octahedroid are magic. In fact no octahedroid of lower order than 8 can have all its plain diagonals magic; but by sacrificing this property we can obtain a 5^4 with many more magic properties than the above.

495	58	271	589	152	478	66	259	597	165	486	54	267	585	173	499	62	255	593	156	482	75	263	576	169
178	391	84	297	615	186	379	92	285	623	199	387	80	293	606	182	400	88	276	619	195	383	96	289	602
511	204	417	110	323	524	212	405	118	306	507	225	413	101	319	520	208	421	114	302	503	216	409	122	315
349	537	230	443	6	332	550	238	426	19	345	533	246	439	2	328	541	234	447	15	336	529	242	435	23
32	375	563	126	469	45	358	571	139	452	28	366	559	147	465	36	354	567	135	473	49	362	555	143	456
70	258	596	164	477	53	266	584	172	490	61	254	592	160	498	74	262	580	168	481	57	275	588	151	494
378	91	284	622	190	386	79	292	610	198	399	87	280	618	181	382	100	288	601	194	395	83	296	614	177
211	404	117	310	523	224	412	105	318	506	207	425	113	301	519	220	408	121	314	502	203	416	109	322	515
549	237	430	18	331	532	250	438	1	344	545	233	446	14	327	528	241	434	22	340	536	229	442	10	348
357	575	138	451	44	370	558	146	464	27	353	566	134	472	40	361	554	142	460	48	374	562	130	468	31
270	583	171	489	52	253	591	159	497	65	261	579	167	485	73	274	587	155	493	56	257	600	163	476	69
78	291	609	197	390	86	279	617	185	398	99	287	605	193	381	82	300	613	176	394	95	283	621	189	377
411	104	317	510	223	424	112	305	518	206	407	125	313	501	219	420	108	321	514	202	403	116	309	522	215
249	437	5	343	531	232	450	13	326	544	245	433	21	339	527	228	441	9	347	540	236	429	17	335	548
557	150	463	26	369	570	133	471	39	352	553	141	459	47	365	561	129	467	35	373	574	137	455	43	356
595	158	496	64	252	578	166	484	72	265	586	154	492	60	273	599	162	480	68	256	582	175	488	51	269
278	616	184	397	90	286	604	192	385	98	299	612	180	393	81	282	625	188	376	94	295	608	196	389	77
111	304	517	210	423	124	312	505	218	406	107	325	513	201	419	120	308	521	214	402	103	316	509	222	415
449	12	330	543	231	432	25	338	526	244	445	8	346	539	227	428	16	334	547	240	436	4	342	535	248
132	475	38	351	569	145	458	46	364	552	128	466	34	372	565	136	454	42	360	573	149	462	30	368	556
170	483	71	264	577	153	491	59	272	590	161	479	67	260	598	174	487	55	268	581	157	500	63	251	594
603	191	384	97	290	611	179	392	85	298	624	187	380	93	281	607	200	388	76	294	620	183	396	89	277
311	504	217	410	123	324	512	205	418	106	307	525	213	401	119	320	508	221	414	102	303	516	209	422	115
24	337	530	243	431	7	350	538	226	444	20	333	546	239	427	3	341	534	247	440	11	329	542	235	448
457	50	363	551	144	470	33	371	564	127	453	41	359	572	140	461	29	367	560	148	474	37	355	568	131

Fig. 5.

In Fig. 5 the great square is magic, nasik and 25-ply: the 25 subsidiaries are purposely not nasik, but they are all magic in rows and columns. If we take up the subsidiaries in the way just described, viz., 513, 221, etc., for the first cube; 205, 413, etc., for the second cube, and so on, we get a 5^4, which has 20 contiguous rectilinear summations through any cell, viz., the 4 directions parallel to the edges and the whole of the 16 three-dimensional diagonals parallel to the great diagonals of any cubical section. If the reader will write out the four aspects of the octahedroid, in the way already explained, he will be able to verify this statement. As an example, the 20 summations through the cell containing the number 325, which lies in the first plate of the first cube of the P_1 aspect, are here shown:

ROW	COLUMN	LINE	"z"	CUBICAL DIAGONALS															
				P_1 ASPECT				P_2 ASPECT				V ASPECT				H ASPECT			
325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325	325
513	8	508	512	534	388	607	3	538	392	611	7	533	387	608	4	413	103	507	509
201	466	216	204	143	576	169	456	126	589	152	469	141	579	166	458	501	406	219	218
419	154	404	416	477	44	451	164	494	31	468	151	479	41	454	162	119	214	401	402
107	612	112	108	86	232	13	617	82	228	9	613	87	233	12	616	207	517	113	111

Since there are 20 magic summations through each of the 625 cells and each summation involves 5 cells, the total number of different symmetrical magic summations in this octahedroid is 2500. This does not include the 8 central hyperdiagonals, which are also magic, for this is not a symmetrical property since *all* the hyperdiagonals are not magic.

The next odd order, 7, was the one which Frost attacked. Glass models of his 7 cubes were for many years to be seen at the South Kensington Museum, London, and possibly are still there. He does not appear to have completely grasped the analogy between magics in 3 and 4 dimensions, and from the account he gives in *The Quarterly Journal,* he evidently assumed that the figure was magic on all its plane diagonals. Actually it is magic on all plane diagonals only in the P-aspect; in the other 3 aspects it is nasik in one set of planes but only semi-nasik in the other two sets of planes, therefore of the 12 plane diagonals through any cell of the octahedroid only 9 are magic.[9] Frost obtained his figure by direct application of the method of paths; the present writer using the method of formative square has obtained an example with one additional plane magic diagonal. It is shown as a great square of order

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[9] Probably the reader will have already noticed that although there are 4 aspects, and 6 plane diagonals appear in each aspect, yet there are only 12 plane diagonals in all, since, with this method of enumeration, each diagonal occurs twice.

49, magic on its 49 rows, 49 columns and 98 diagonals, and 49-ply, that is *any* square bunch of 49 numbers gives the same sum as a row or column. The 49 subsidiaries are equally weighted nasiks, magic on their 7 rows, 7 columns and 14 diagonals. If the subsidiaries be taken up along the Indian paths, as in the previous examples, we get 7 cubes forming an octahedroid of order 7. This is magic on the 4 directions parallel to the edges, is completely plane nasik in the P_1 and P_2 aspects, and in the other two aspects it is nasik in two sets of planes and crude in the third set. Therefore of the 12 plane diagonals through any cell 10 are magic. It is practically certain that we can go no further in this direction with this order, but by giving up the magic plane diagonals we can, as with 5^4 above, obtain a larger number of magic summations on the higher diagonals.

When we consider the even orders we find those $\equiv 2 \pmod 4$ of little interest. The powerful methods used for the other orders are now useless if we insist on using consecutive numbers: we must employ other methods. Professor Kingery's Fig. 4 cannot be made into a magic octahedroid by any shuffling of its subsidiaries. The best methods here, are either to use an extension of Thompson's method of pseudo-cubes, as employed by Mr. Worthington in his construction of 6^3 (*The Monist,* XX, pp. 303-309),[10] or, best of all, to use the method of reversions.

With orders $\equiv 0 \pmod 4$ we can give a greater number of ornate features than with any other orders. We quote one example below (Fig. 6).

The columns of Fig. 6 give the 4 cubes of an octahedroid of order 4, which is crude in plane diagonals, but is magic on every other contiguous rectilinear path, it has therefore 28 such paths through each cell. The 28 magic paths through the cell containing the number 155 are displayed below.

ROW	COLUMN	LINE	“z”	CUBICAL DIAGONALS															
				P_1 ASPECT				P_2 ASPECT				V ASPECT				H ASPECT			
155	155	155	155	155	155	155	155	155	155	155	155	155	155	155	155	155	155	155	155
38	70	98	101	2	50	242	194	5	53	245	197	77	125	113	65	36	33	225	228
91	171	151	154	103	103	103	103	106	106	106	106	166	166	166	166	86	86	86	86
230	118	110	104	254	206	14	62	248	200	8	56	116	68	80	128	237	240	48	45

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[10] It was by this method that Firth in the 80's constructed what was, almost certainly, the first correct magic cube of order 6. *Mr. Worthington's introduction of magic central diagonals on all the faces is new. Though, of course, not a symmetrical summation, this is a very pleasing feature.*

HYPERDIAGONALS							
155	155	155	155	155	155	155	155
256	208	16	64	253	205	13	61
102	102	102	102	102	102	102	102
1	49	241	193	4	52	244	196

1	128	193	192
240	145	48	81
49	80	241	144
224	161	32	97

254	131	62	67
19	110	211	174
206	179	14	115
35	94	227	158

4	125	196	189
237	148	45	84
52	77	244	141
221	164	29	100

255	130	63	66
18	111	210	175
207	178	15	114
34	95	226	159

248	137	56	73
25	104	217	168
200	185	8	121
41	88	233	152

11	118	203	182
230	155	38	91
59	70	251	134
214	171	22	107

245	140	53	76
28	101	220	165
197	188	5	124
44	85	236	149

10	119	202	183
231	154	39	90
58	71	250	135
215	170	23	106

13	116	205	180
228	157	36	93
61	68	253	132
212	173	20	109

242	143	50	79
31	98	223	162
194	191	2	127
47	82	239	146

16	113	208	177
225	160	33	96
64	65	256	129
209	176	17	112

243	142	51	78
30	99	222	163
195	190	3	126
46	83	238	147

252	133	60	69
21	108	213	172
204	181	12	117
37	92	229	156

7	122	199	186
234	151	42	87
55	74	247	138
218	167	26	103

249	136	57	72
24	105	216	169
201	184	9	120
40	89	232	153

6	123	198	187
235	150	43	86
54	75	246	139
219	166	27	102

Fig. 6.

But this does not exhaust the magic properties, for this figure is 4-ply in every plane section parallel to any face of the octahedroid. If the reader will examine the figure in its four aspects he will find that 6 such planes can be drawn through any cell, and since a given number is a member of four different 4-ply bundles in each plane, it follows that each number is a member of 24 different bundles. If we add the 28 rectilinear summations through any cell we see that each of the 256 numbers takes part in 52 different summations. The total number of different magic summations in the octahedroid is therefore $\frac{256 \times 52}{4} = 3328$. The six planes parallel to the faces through 155 are shown in Fig. 7, and from them the 24 different bundles in which 155 is involved can be at once determined.

11	118	203	182
230	155	38	91
59	70	251	134
214	171	22	107

19	110	211	174
230	155	38	91
31	98	223	162
234	151	42	87

131	118	143	122
110	155	98	151
179	70	191	74
94	171	82	167

25	104	217	168
230	155	38	91
28	101	220	165
231	154	39	90

137	118	140	119
104	155	101	154
185	70	188	71
88	171	85	170

145	110	148	111
104	155	101	154
157	98	160	99
108	151	105	150

Fig. 7.

The reader might object that the border cells of a square section cannot be involved in 4 bundles of that section; but this would be to overlook the congruent property. The number 107, which occupies a corner cell of the first section given above is contained in the following bundles:

251	134
22	107

134	59
107	214

22	107
203	182

107	214
182	11

It is noticeable that the four corner cells of a square form one of its 4-ply bundles.

It would have been desirable to indicate the methods by which the above examples have been constructed, but exigencies of space forbid. The four orders dealt with, 3, 5, 7, 4, were all obtained in different ways. Fig. 6 was constructed by direct application, in four dimensions, of the method of paths; in fact, it is the octahedroid

$$\left|\begin{matrix}2&2&2&1\\2&2&1&2\\2&1&2&2\\1&2&2&2\end{matrix}\right| 4.$$

The whole of its magic properties may be deduced by examination of the determinant and its adjoint, without any reference to the constructed figure. There is therefore nothing empirical about this method.

C. PLANCK.

HAYWARD'S HEATH, ENGLAND.

BJOERKLUND'S "DEATH AND RESURRECTION."

The résumé and discussion of the above book* with reference to and contrast with the writings and position of Dr. Carus, by Mr. J. E. Fries in *The Monist,* April issue, is deeply interesting. The book itself is absorbingly so, evidencing large knowledge in research and much independent thinking. It may not be conceded, however, as convincing in establishment of the thesis in theory—the soul as a thing or matter of the cells. The author's discussion scarce touches the soul, stopping short of the thing in pursuit, but with an incompetent substitute. Cells are given individual but not *independent* existence, and on that ground alone the cell is incompetent as basic to the soul.

Mr. Fries begins with the "life-force," and the Editor's earlier published article "Life and the Soul," and the point in antagonism with Dr. Carus is emphasized that "life has no roots whatever in the material world." Surely "a strange statement" this! Dr. Carus is quoted as follows: "Living bodies consist of the very same materials of which the rest of the world is composed," which physics and chemistry make certain. The "life phenomenon is *sui generis*"; this, too, science concedes. Again "...life is a manifestation of energy which forms a category of its own" etc., a not pronounced materialism this, surely. Here, then, is life-force *sui generis* yet natural, no mysticism or supernaturalism involved or required.

To the somewhat equivocal question, "With no other resources than the material world to fall back upon, how then did life grow out of matter and physical energy?" it is answered: "Surely we must look beyond the boundaries of the visible world." To this it may be replied: Energy is not of category with matter and the visible. Within the cosmic world, however, are found both matter and energy, and whatever other factor may be essential to the "origin and substance of life." The cosmical is the natural, and

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* *Death and Resurrection from the Point of View of the Cell-Theory* by Gustaf Björklund. Translated from the Swedish by J. E. Fries. Chicago, Open Court Pub. Co., 1910.